POSSIBILITIES OF MHD EFFECT ON THE LOCALLY IONIZED FLOW PAST A CIRCULAR CYLINDER

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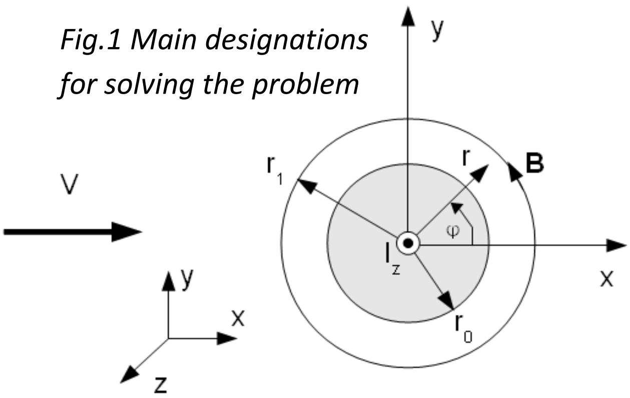
Problems of magneto-hydrodynamic interaction on conducting flow around cylinder with current through its axis, which forms magnetic field were considered numerically in works [1-2], experimentally in works [3-5], and numerically and experimentally in works [6]. The main purpose in these works was to decrease the heat flux on surface by means of MHD interaction with the flow.

Results from numerical calculation [1-2] predicted dramatic decrease (in orders) of heat flux due to MHD interaction. However, experimental results showed much lower effect (decrease of heat flux is

Fig.1 Main designations for solving the problem



interaction zone, which, according to [7], strongly influents on the effectiveness of MHD flow control. Analysis of obtained analytic solution allows one to understand the features of MHD interaction with flow around cylinder and gives the possibility to optimize MHD flow control.

Main equations and their solutions

Consider a cylinder with radius r_0 , around which the magnetic field is formed by current passing through the axis of the cylinder. The ionized zone is placed around the cylinder and is restricted in the interval of radius $r_0 \le r \le r_1$. The scheme and main designations of symbols used for solving the problem are showed in fig.1. Using general Ohm's law

$$\mathbf{j} + \mu_e (\mathbf{j} \times \mathbf{B}) = \sigma \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (1)$$

and law of continuity of current $\nabla \cdot j = 0$, in work [8] the differential equation for electrical potential Φ in cylindrical coordinate system was obtained:

$$\left(r^2 + \beta_0^2 r_0^2\right)^2 \frac{\partial^2 \Phi}{\partial \varphi^2} + r^4 \left(r^2 + \beta_0^2 r_0^2\right) \frac{\partial^2 \Phi}{\partial r^2}$$
$$+ r^3 \left(r^2 + 3\beta_0^2 r_0^2\right) \frac{\partial \Phi}{\partial r} + r\left(r^2 - \beta_0^2 r_0^2\right) \frac{\beta_0^2 r_0^2}{\mu_e} \cdot v_r$$
$$-\left(r^4 + \beta_0^2 r^2 r_0^2\right) \frac{\beta_0^2 r_0^2}{\mu_e} \frac{\partial v_r}{\partial r} - 2r^2 \beta_0^3 r_0^3 E_z = 0 \qquad (2)$$

where j – current density, B - magnetic field, whose value is related to radius by the following way: $B = B_0 r_0 / r$, E - electric field, v - flow velocity, μ_e - mobility of electron, σ - conductivity of the gas, $\beta_0 = \mu_e B_0$ – characteristic value of Hall parameter at surface of the cylinder, v_r - radial velocity of the flow, E_z - external electric field along axis z. Mention that in equation (2) we considered the conductivity of flow in the ionized zone as constant.

Such an equation can be solved numerically for arbitrary distribution of radial velocity of the flow. Analytic solution of this equation can be obtained only under very simple approximations. In work [8] the approximation of potential flow was applied, for which:

$$v_r = V_0 \cdot \cos\varphi \cdot \left(1 - \left(r_0/r\right)^2\right) \qquad (3)$$

where V_0 – starting speed upstream.

The use of such an approximation allows one to obtain analytic expression for the distribution of potential $\Phi(r,\varphi)$ in the MHD interaction zone.

$$\Phi_1\left(r,\varphi\right) = -V\cos\varphi\frac{r^2+r_0^2}{\mu_e r} + E_z\frac{\left(\beta_0 r_0\right)^3}{2r^2} \qquad (4)$$
$$+C_1^1 Z_1\left(1,\beta_0 r_0/r\right) + C_2^1 Z_2\left(1,\beta_0 r_0/r\right)$$

where functions $Z_1(k,\beta_0 r_0/r), Z_2(k,\beta_0 r_0/r)$ are expressed via modified Bessel functions I and K by the following way:

$$Z_1\left(k,x\right) = I_k\left(kx\right) + xI_{k+1}\left(kx\right)$$
$$Z_2\left(k,x\right) = K_k\left(kx\right) - xK_{k+1}\left(kx\right)$$

Constants C_1^1, C_2^1 are determined by the boundary conditions at the edge of MHD interaction zone r_0 and r_1, and have different values for cases of conductive and nonconductive surface of cylinder. In the case of nonconductive surface:

$$C_1^1 = -\frac{\left(r_1 - r_0^2/r_1\right)K_1\left(\beta_0\right)}{I_1\left(\beta_0 r_0/r_1\right)K_1\left(\beta_0\right) - I_1\left(\beta_0\right)K_1\left(\beta_0 r_0/r_1\right)}$$
$$C_2^1 = \frac{\left(r_1 - r_0^2/r_1\right)I_1\left(\beta_0\right)}{I_1\left(\beta_0 r_0/r_1\right)K_1\left(\beta_0\right) - I_1\left(\beta_0\right)K_1\left(\beta_0 r_0/r_1\right)}$$

(5)

While in the case of conductive surface:

$$C_1^1 = -\frac{2K_1\left(\beta_0 r_0/r_1\right)r_0 + \left(r_1 - r_0^2/r_1\right)Z_2\left(1,\beta_0\right)}{Z_2\left(1,\beta_0\right)I_1\left(\beta_0 r_0/r_1\right) - Z_1\left(1,\beta_0\right)K_1\left(\beta_0 r_0/r_1\right)}$$
$$C_2^1 = \frac{2I_1\left(\beta_0 r_0/r_1\right)r_0 + \left(r_1 - r_0^2/r_1\right)Z_2\left(1,\beta_0\right)}{Z_2\left(1,\beta_0\right)I_1\left(\beta_0 r_0/r_1\right) - Z_1\left(1,\beta_0\right)K_1\left(\beta_0 r_0/r_1\right)}$$

(6)

The existence of analytic expression for potential $\Phi(r,\varphi)$ allows us to obtain analytic expressions for the distribution of current in the MHD interaction zone, and as a result, the distribution of Lorentz force acting on the flow.

$$j_r = \frac{\sigma B_0 V}{\beta_0}\cdot\left[\begin{array}{l}\frac{1}{r}\left(C_1^1 I_1\left(\frac{\beta_0 r_0}{r}\right) + C_2^1 K_1\left(\frac{\beta_0 r_0}{r}\right)\right)\\ +1-\left(\frac{r_0}{r}\right)^2\end{array}\right]\cdot\cos\varphi$$
$$j_z = \sigma\cdot\left(E_z + B_0\frac{r_0}{r}\cdot v_r\right) - j_r\cdot\beta_0\cdot\frac{r_0}{r}$$

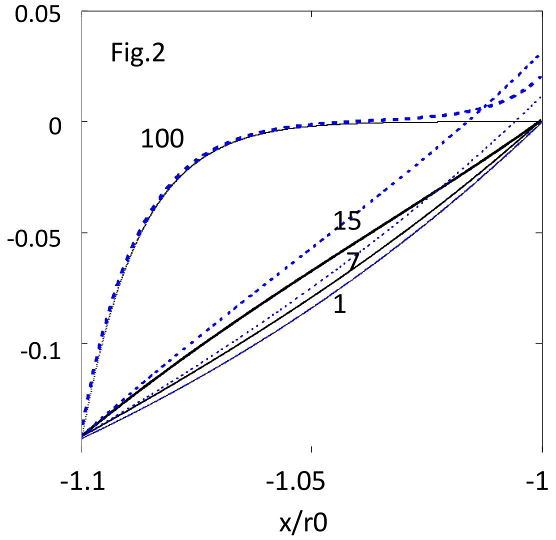
(7)

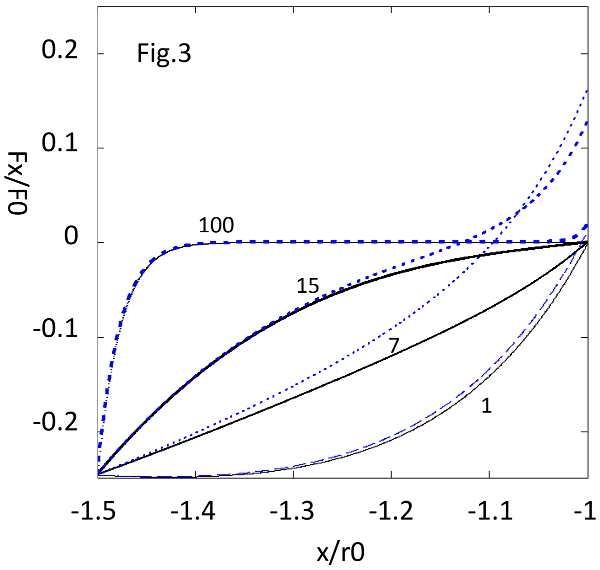
$$F_r = -j_z \cdot B_\varphi \qquad (8)$$

where j_r, j_z, F_r - density of radial current, density of current along axis z, and radial Lorenz force in unit volume, respectively

Results of calculation

In fig.2-4 are shown the distribution of Lorentz force $F_x = F_r \cos\varphi$, which is normalized to the value $F_0 = \sigma B_0^2 V_0$, with different Hall parameters β_0 and relative values of the radius $\tilde{r}_1 \equiv r_1 / r_0$ of outer edge of MHD interaction zone. Both results for conductive(dashed lines) and nonconductive(solid lines) surfaces of cylinder are shown. Mention that for the case considered, negative value of the force corresponds to deceleration of flow, while positive one – acceleration.





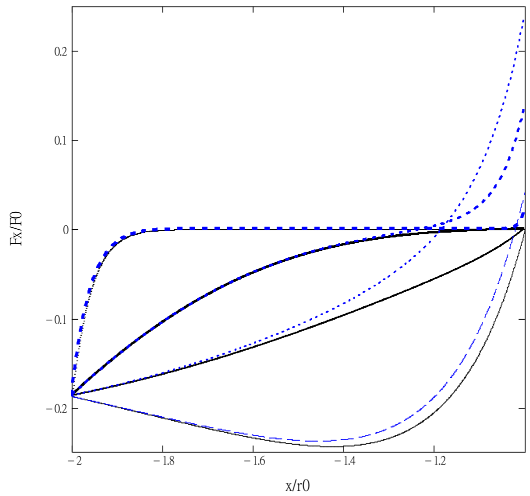


Fig.2-4 Lorenz force acting along deceleration line ($\varphi = \pi$) in MHD interaction zone $1 \le r / r_0 \le \tilde{r}_1$ for different Hall parameters.

Solid lines for nonconductive surface, dashed lines – conductive surface.

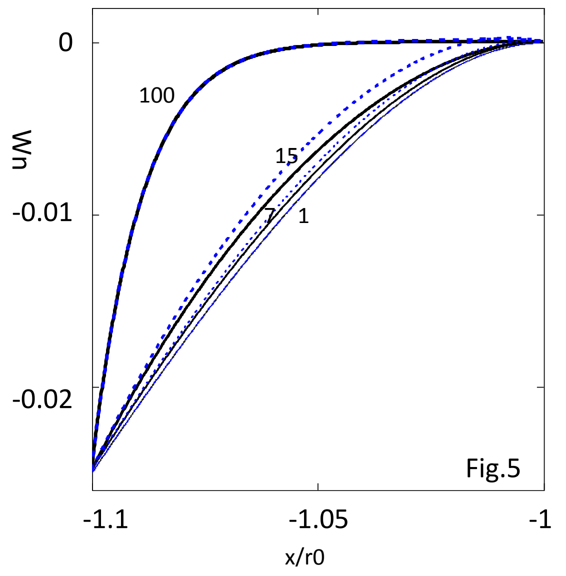
the results showed in the figures one can

1.

effectiveness of deceleration decreases with increase of Hall parameter.

2. Obtained results showed that the decelerating effect of Lorentz force is more apparent in the case of nonconductive surface of cylinder. The main influence of the type of surface on the Lorentz force can be observed near by the surface of cylinder. Under some conditions, Lorentz force near by the surface accelerates the flow. Also, the influence of the type of surface on Lorentz force achieves maximum with intermediate values of Hall parameters. For example, in fig.3-4 the maximum difference of the value of Lorentz force due to different type of surface is obtained at $\beta_0 \approx 7$.

Let us introduce the work done by Lorentz force in unit time and unit volume as $W = F_r \cdot v_r$. In fig.5-7 are showed distribution of $W_n = W / (\sigma B_0^2 V_0 r_0)$ in MHD interaction zone $1 \le r / r_0 \le \tilde{r}_1$ at $\varphi = \pi$ for different Hall parameters $\beta_0 = 1, 7, 15, 100$, $\tilde{r}_1 \equiv r_1 / r_0 = 1.1, 1.5, 2$, for the case of conductive (dashed lines) and nonconductive(solid lines) surface of cylinder. One can see that the relationship of Wn(x), principally, is similar to that of Lorentz force. The main difference is observed in the case of conductive surface. In this case Lorentz force at the surface has large enough positive value, while the value of W at the surface of cylinder turns to zero. Obviously, it is because of the zero radial velocity of flow at the surface of cylinder.



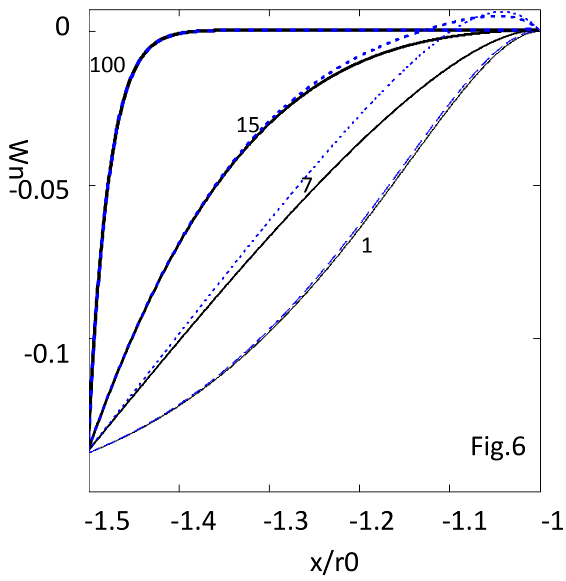


Fig.5-7 Normalized density of work W_n . Other conditions are the same as fig.2-4

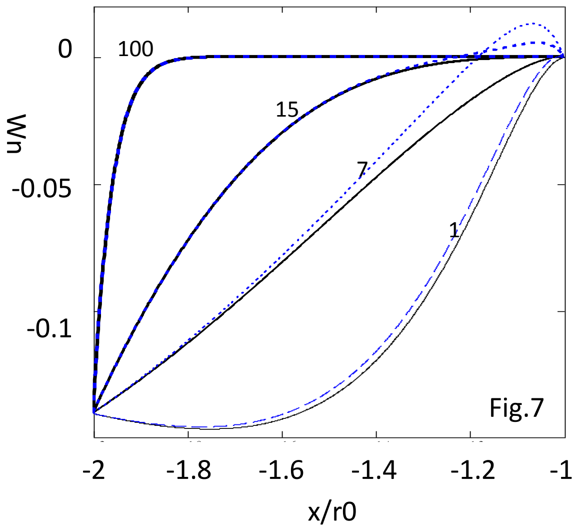


Fig.7

In fig.8-10 showed the distribution of $W_n = W/(\sigma B_0^2 V_0 r_0)$ at $\varphi = \pi$ in the interval $1 \le r/r_0 \le 1.5$ with different external field $E_z/(B_0 V_0) = -0.1, 0, 0.1$ at $\beta_0 = 1, 7, 20$. These figures show the possibility to control MHD interaction by means of external electric field. Mention that negative external field leads to increase the effect of flow deceleration by MHD interaction.

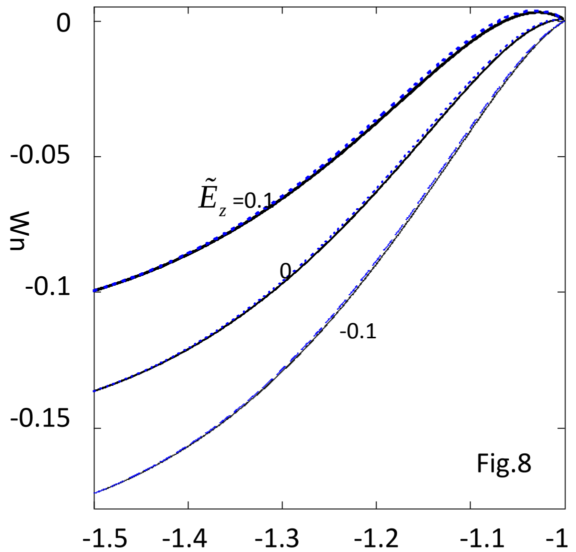


Fig.8

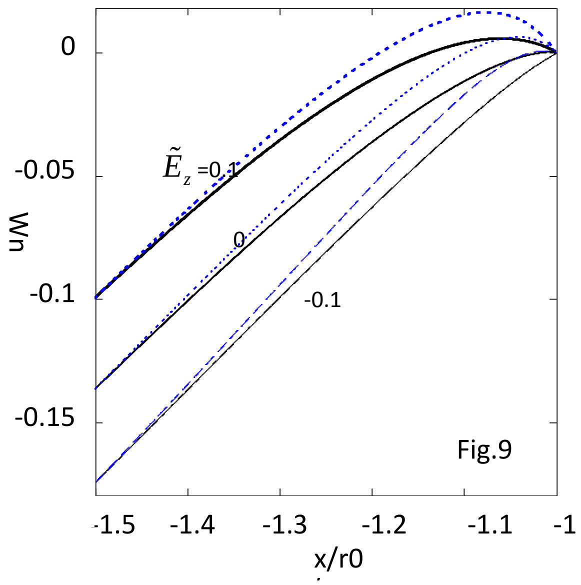


Fig.9

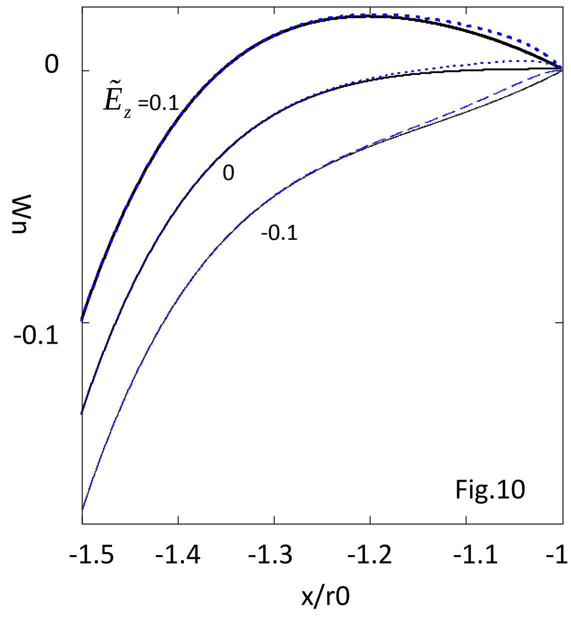


Fig.10

Fig.8-10 Relation of W_n from radius $1 \le r/r_0 \le 1.5$. With different external field $\tilde{E}_z = E_z/(B_0 V_0) = -0.1, 0, 0.1$

Hall parameters in fig.8-10 are 1,7,20, respectively.
Solid lines for case of nonconductive surface, while dashed lines – conductive surface.

The most interesting point relating to flow control is the total work done by Lorenz force in the MHD interaction zone. Determine the work in unit cross section, which is done by Lorenz force along deceleration line $(\varphi = \pi)$ as following:

$$A \equiv \int_{r0}^{r1} W dr$$

$$A_n \equiv \frac{A}{W_0(r1 - r0)} = \frac{1}{\tilde{r}_1 - 1}\int_{1}^{\tilde{r}1} W_n d\tilde{r}$$

This parameter gives clear concept about the influence of MHD interaction on the flow, and it allows one to estimate the effectiveness of the influence.

In fig.11 showed A_n for different width of ionized zone $1 \le \tilde{r}_1 \le 5$, with $\beta_0 = 1, 7, 20$ and $E_z = 0$.

Results shown in this figure show that the value of parameter An depends to the type of surface of cylinder, Hall parameter, and width of MHD interaction zone. For any given Hall parameter exists some optimized width of MHD interaction zone, under which the value of An achieves minimum. By this way, the effectiveness of MHD flow control for locally ionized flow is essentially more effective than that for whole ionized flow.

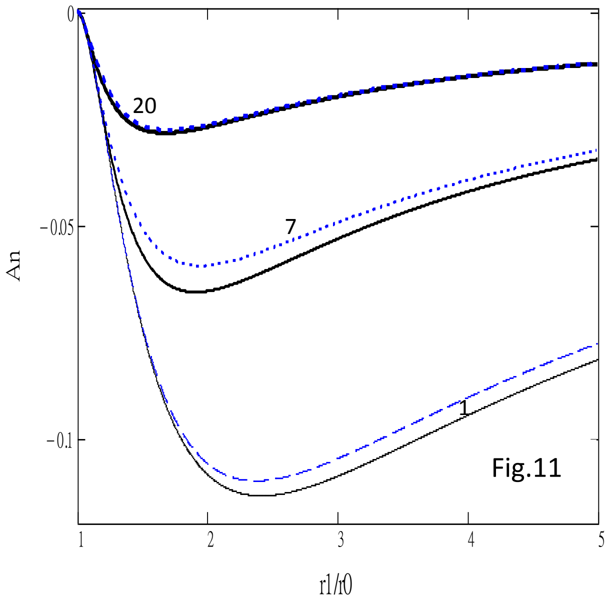


Fig.11 Relation of A_n from width of MHD zone, at different Hall parameters (1,7, 20). Solid lines for nonconductive surface, dashed lines for conductive surface.

In fig.12 showed relation of A_n from Hall parameter, in the case of $E_z/(B_0V_0) = -0.1, 0, 0.1$ and $\tilde{r}_1 = 1.5$.

Results shown in this figure show that external field allows one to essentially control the process of MHD interaction with flow, and the increase of Hall parameter leads to decrease the effectiveness of flow deceleration by MHD interaction. Moreover, mention that the type of surface of cylinder essentially influents MHD interaction. In the case of nonconductive surface of cylinder, the deceleration of flow due to MHD interaction is more apparent than that in the case of conductive surface.

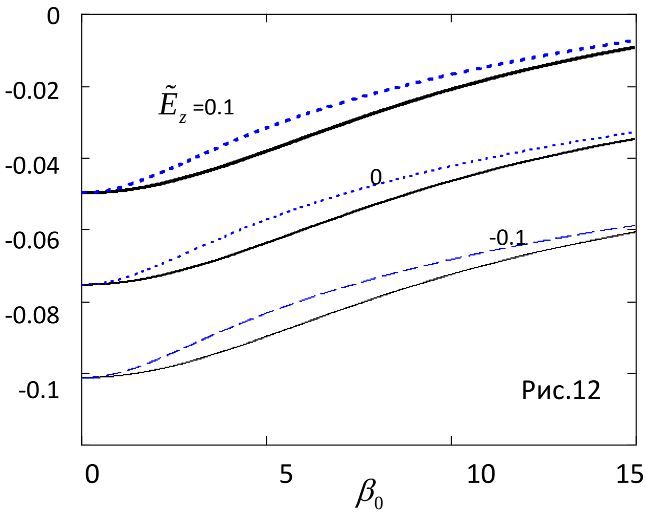


Fig.12 Relation of A_n from Hall parameter (1-15). At different external field.

Conclusion

The use of analytic solution for electric potential in nonuniform magnetic field in locally ionized flow around cylinder allows one to analyze the physical processes of MHD interaction with the flow. Obtained results show that the type (conductive or nonconductive) of surface of cylinder has influence on MHD interaction. The use of external electric field allows one to essentially influent the MHD effect. One very important parameter, determining the effectiveness of MHD interaction with flow, is the width of MHD interaction zone. There exists some optimized width of MHD interaction zone, which depends on the Hall parameter. By this way, the negative influence of the Hall parameter on MHD deceleration process may be minimized by means of correctly choice of width of MHD interaction zone. The effectiveness of MHD flow control for locally ionized flow will be much higher than that for wholly ionized flow. The obtained results hint out the possible ways to optimize MHD control of flow around cylinder.

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