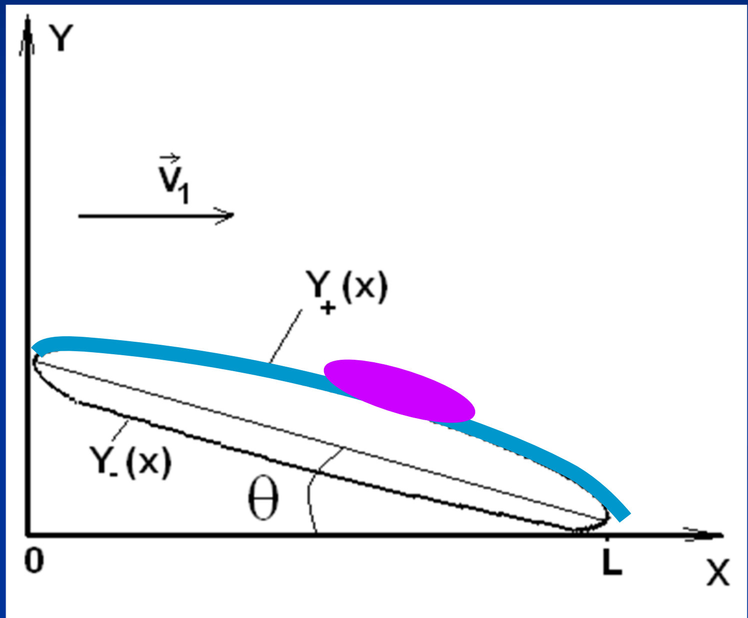
The research of subsonic flow around thin profile with external influence



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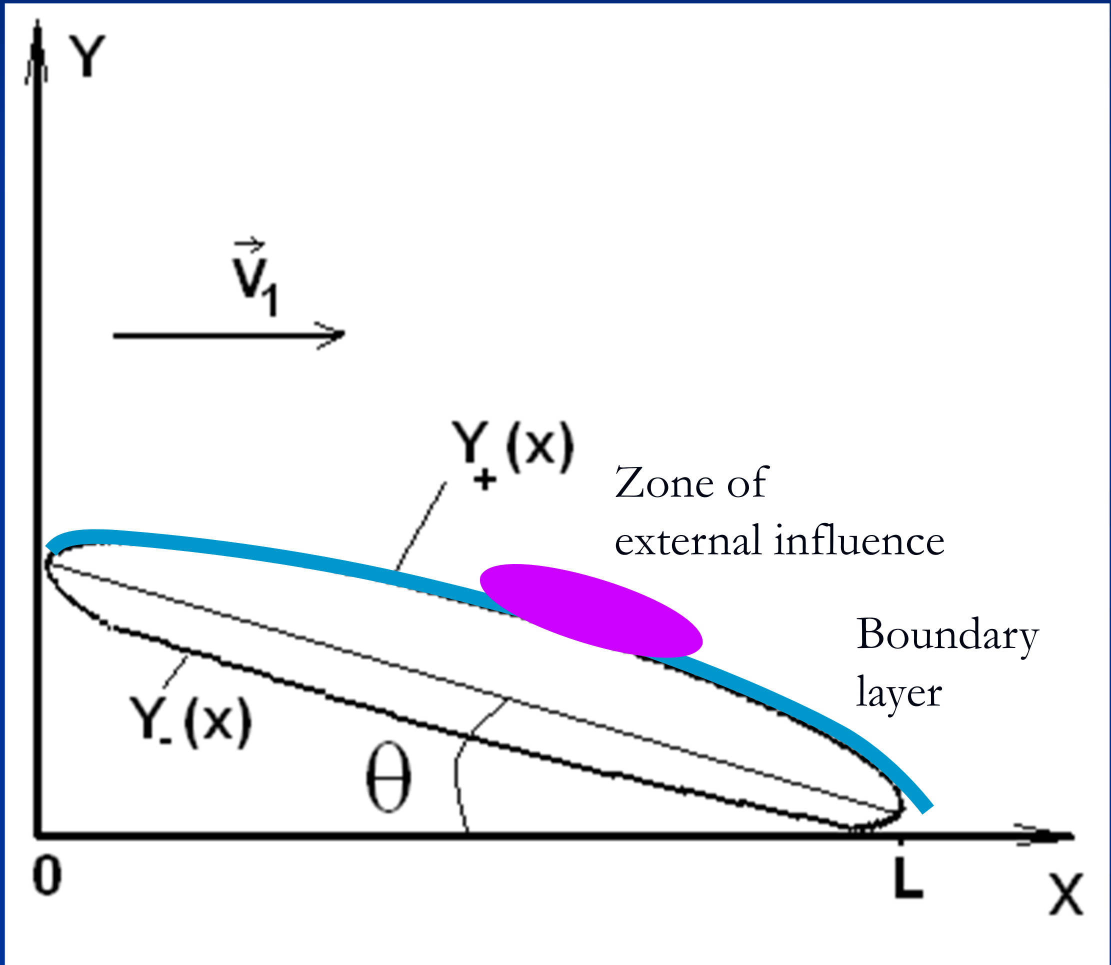
Introduction

- Many previous works showed that plasma energy deposition helps one to control flow and to optimize aerodynamic performance of aircrafts.
- To use locally ionized plasma near by the surface is more reliable due to less power consumption to produce and to maintain plasma.
- In this work the boundary flow of a thin profile with small AOA and with surface plasma is concerned. The analytical solution was found.

Plan of report

- The model and its solution.
- Use given external influence to find the resulted disturbance.
- Use expected disturbance to find necessary external influence.

Thin profile at small AOA with thin layer of ionized gas



- Thin profile
- Small AOA
- Thickness of zone of external influence is much larger than that of boundary layer.
- Thickness of zone of external influence is much less than L.

$$B_0{}^2\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=\frac{(\gamma-1)Q(x,y)}{\rho_0 c^2}-\frac{\gamma MF(x,y)}{\rho_0 c}\equiv\frac{\Phi_1(x,y)}{L^2}$$

$$B_0^2=1-M_0^2$$

$$u(\overline{x})=\frac{1}{L}\frac{\partial\Phi(\overline{x},\overline{y})}{\partial\overline{x}}\bigg|_{\overline{y}=\overline{Y}_+(\overline{x})},v(\overline{x})=\frac{1}{L}\frac{\partial\Phi(\overline{x},\overline{y})}{\partial\overline{y}}\bigg|_{\overline{y}=\overline{Y}_+(\overline{x})},\frac{1}{P_0}\frac{d\Delta P}{d\overline{x}}=-\frac{\gamma M^2}{L}\frac{\partial^2\Phi(\overline{x},\overline{y})}{\partial^2\overline{x}}\bigg|_{\overline{y}=\overline{Y}_+(\overline{x})}$$

If external influence is independent to y:

$$u(\overline{x})=\frac{cM}{\pi B_0}\int_0^1\overline{Y}'_+(\xi)\frac{(\overline{x}-\xi)}{\sqrt{(\overline{x}-\xi)^2+\overline{y}^2}}d\xi-\frac{1}{2\pi B_0{}^2 L}\int_0^1\overline{\Phi}_1(\xi)G(\overline{x},\xi)d\xi$$

If thickness of plasma is much less than that of profile:

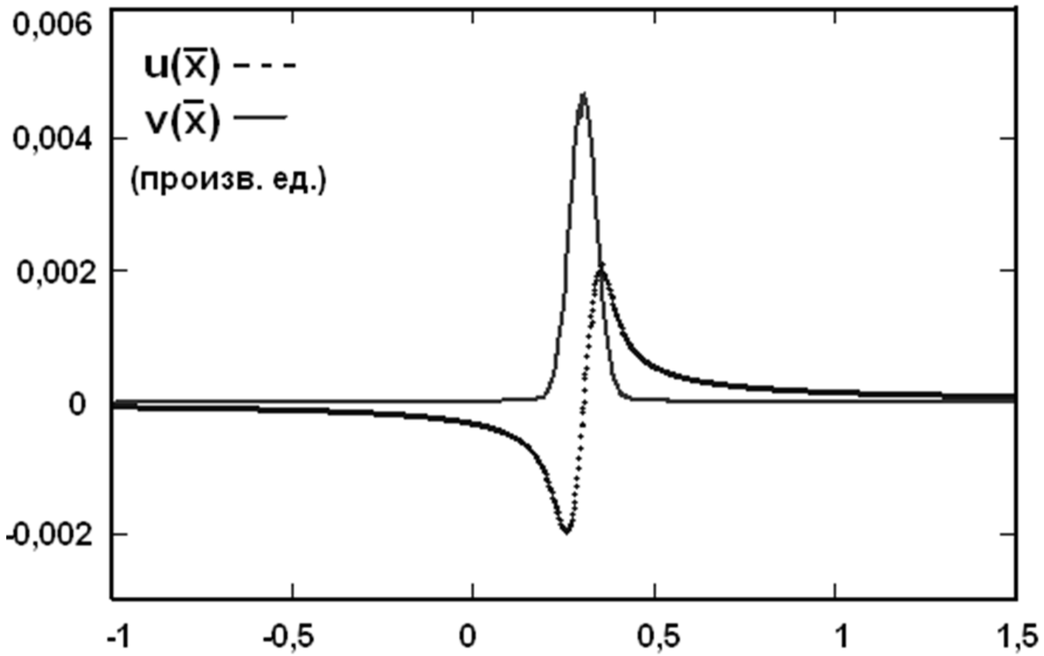
$$u(\overline{x})=-\frac{cM}{\pi B_0}\int_0^1\overline{Y}'_+(\xi)\frac{1}{(\overline{x}-\xi)}d\xi-\frac{1}{2\pi B_0{}^2 L}\int_0^1\overline{\Phi}^+1(\xi)G(\overline{x},\xi)d\xi$$

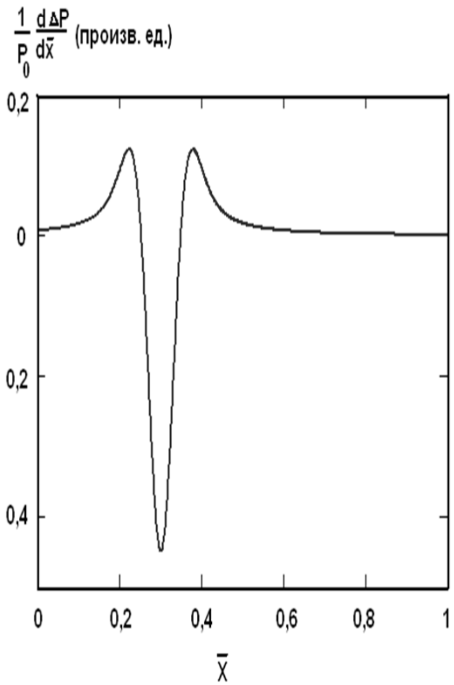
$$G(x,\xi)=-arctg\left(\frac{\overline{\delta}_{pl}B_0}{\overline{x}-\xi}\right)$$

How to use the solution

- Use given external influence to find the resulted disturbance.
- Use needed disturbance to find necessary external influence.
- Find the effective profile by considering external influence.

Use given external influence to find the resulted disturbance





- M=0.7, AOA=0
- Gaussian heating source at 0.3 , width of 0.05
- Because of heating, there will be transverse velocity near the heating source (the flow is expanded).
- Due to flow expansion the longitudinal velocity will decrease at first, and the pressure increases.
- Then due to gas escaping, the pressure near to the source will decrease, which leads to accelerate the flow.
- Besides small zone of pressure increasing, there is a relatively larger zone of pressure decreasing, which helps to prevent flow separation.

Use needed disturbance to find necessary external influence.

If needed pressure disturbance is given:

$$\frac{1}{P_0}\Delta P_{plasma} = f(x)$$

The necessary external influence $\overline{\Phi}_1(x)$ can be found by solving the following equation:

$$\frac{\gamma M}{2\pi c B_0{}^2 L}\int_0^1 \overline{\Phi}_1(\xi) G(\overline{x},\xi)\,d\xi = f(x)$$

$$G(x,\xi) = -arctg\left(\frac{\overline{\delta}_{pl} B_0}{\overline{x}-\xi}\right)$$

$$G_1(\overline{x},\xi) = \frac{2\overline{\delta}_{pl} B_0}{\xi - \overline{x}}$$

For case of whole surface:

$$\overline{\Phi}_1(\overline{x}) = -\frac{2B_0 Lc}{\gamma\pi M\overline{\delta}_{pl}}\sqrt{\frac{(1-\overline{x})}{\overline{x}}}\int_0^1 \sqrt{\frac{\xi}{1-\xi}}\, f(\xi)\frac{d\xi}{(\xi-\overline{x})}$$

For case of local optimization at $x \in [a, a+\varepsilon]$

$$\overline{\Phi}_1(\overline{x}) = -\frac{2B_0 Lc}{\gamma\pi M\overline{\delta}_{pl}}\frac{f(a)}{(a-\overline{x})}\sqrt{\frac{(1-\overline{x})a}{\overline{x}(1-a)}}\left[\varepsilon + O(\varepsilon^2)\right]$$

Power consumption is $W = W_Q + W_F$

$$W_Q = \left|\frac{\gamma\overline{\delta}_{pl} P_0}{\gamma - 1}\int_0^1 \left[\overline{\Phi}_1{}^+(x) + \overline{\Phi}_1{}^-(x)\right]_{F=0} dx\right|$$

$$W_F = \left|\frac{\overline{\delta}_{pl} P_0}{M}\int_0^1 \left[\overline{\Phi}_1{}^+(x) + \overline{\Phi}_1{}^-(x)\right]_{Q=0} dx\right|$$

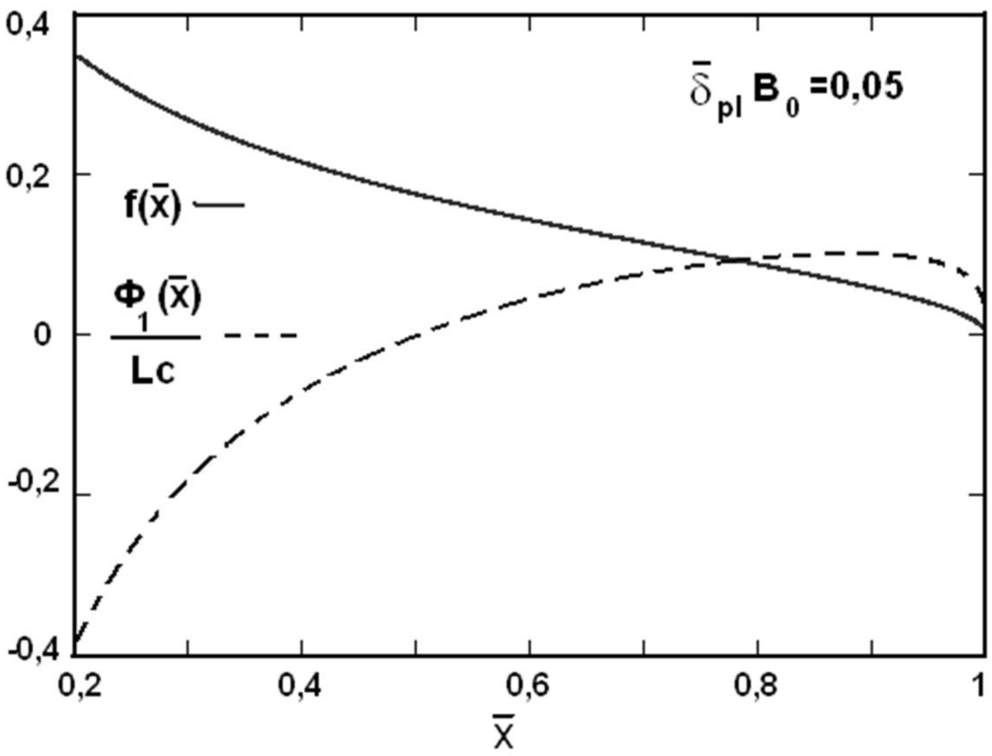
For example, for a plate at small AOA, the pressure disturbance on the upper surface without external influence is

$$\frac{\Delta P}{P_0} = -\theta\gamma M^2 \sqrt{\frac{1-\bar{x}}{\bar{x}}}$$

In order to compensate it, the necessary external pressure disturbance is

$$f(\bar{x}) = \theta\gamma M^2 \sqrt{\frac{1-\bar{x}}{\bar{x}}}$$

Then the necessary external influence:



$$\bar{\Phi}_1(\bar{x}) = -\frac{2B_0\theta MLc}{\pi\bar{\delta}_{p0}}\sqrt{\frac{(1-\bar{x})}{\bar{x}}}\ln\left(\sqrt{\frac{(1-\bar{x})}{\bar{x}}}\right)$$

for whole optimization at $x \in [0, L]$

$$\bar{\Phi}_1(\bar{x}) = -\frac{2B_0\theta MLc}{\pi\bar{\delta}_{p0}}\frac{1}{(a-\bar{x})}\sqrt{\frac{(1-\bar{x})}{\bar{x}}}\left[\varepsilon + O(\varepsilon^2)\right]$$

for local optimization at $x \in [a, a+\varepsilon]$

Find the effective profile by considering external influence.

As showed above, for thin profile, disturbance of longitudinal velocity approaches to :

$$u(\bar{x}) = -\frac{cM}{\pi B_0}\int_0^1 \bar{Y}'_+(\xi)\frac{1}{(\bar{x}-\xi)}d\xi - \frac{1}{2\pi B_0{}^2 L}\int_0^1 \bar{\Phi}^+1(\xi)G(\bar{x},\xi)d\xi$$

When we replace $G(x,\xi) = -arctg\left(\frac{\bar{\delta}_{pl}B_0}{\bar{x}-\xi}\right)$ by $G_1(\bar{x},\xi) = \frac{2\bar{\delta}_{pl}B_0}{\xi-\bar{x}}$

$$u(\bar{x}) = -\frac{cM}{\pi B_0}\int_0^1 \bar{Y}'_+(\xi)\frac{1}{(\bar{x}-\xi)}d\xi - \frac{cM}{\pi B_0}\int_0^1 \frac{\bar{\delta}_{pl}\bar{\Phi}_1^+(\bar{x})}{LcM}\frac{1}{\xi-\bar{x}}d\xi$$

$$= -\frac{cM}{\pi B_0}\int_0^1 \left(Y'_+(x) + \frac{\bar{\delta}_{pl}\bar{\Phi}_1^+(\bar{x})}{LcM}\right)\frac{1}{\xi-\bar{x}}d\xi$$

The two terms can be combined in one term, just like that there is only “effective profile”:

$$\tilde{Y}'_+(\bar{x}) = Y'_+(\bar{x}) + \frac{\bar{\delta}_{pl}\bar{\Phi}_1^+(\bar{x})}{LcM}$$

Which means that the external influence “changes” the form of profile.

Conclusions

- In this work, the analytical solution of the behavior of subsonic boundary flow of a thin profile at small AOA with external energy or force action is obtained. Thanks to the analytical solution, we can
- Use given external actions to find resulted flow behavior (distribution of pressure and etc.)
- Find necessary external actions for some expected flow behavior.
- To calculate the power consumption of external actions.
- To find the effective profile by considering external actions.

~End~

Thank you for attention!